D. C. THREE WIRE ELECTRICAL POWER TRANSMISSION NETWORK AS SOLVED BY RELAXATION METHOD

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ABSTRACT. This paper reveals how the relaxation method can be suitably applied to determine some important quantities such as line currents in D. C. Three Wire Transmission System. The equivalent circuit of this network system is used and the problem is solved easily by applying the principle of relaxational solution of D.C. network, considering the heating effects of steady currents flowing in it. The values of the required quantities thus obtained are compared with those found out by the normal method of network analysis.

INTRODUCTION

D.C. Three Wire Transmission system (Starr, 1946) is used for having considerable economy in feeders and distributors, when the electrical energy to be supplied is fairly large. Although there are different methods for solving this problem the relaxation method proves to be of advantage for yielding many useful informations simultaneously.

In this problem an unbalanced D.C. Three Wire Transmission system, that means the outer lines carrying unequal currents resulting a flow of current in the neutral line, is considered as shown in Fig. 1. The equivalent circuit diagram can be conveniently drawn indicated in Fig. 2, which is then solved by the relaxation method.

First of all Southwell and Black (1938), and later on Dutta (1966), applied the relaxation method in the problem of D.C. networks and showed its usefulness in solving the network problem represented by Fig. 2.

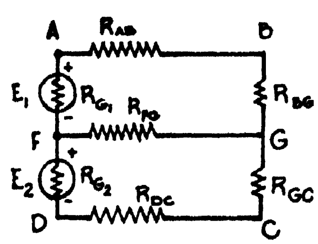

Fig. 1. **Diagram for D.C. three wire Transmission-Network.**

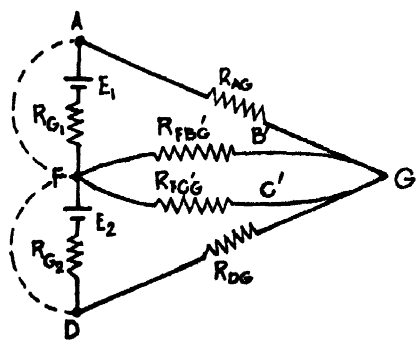

Fig. 2. **Equivalent Circuit diagram for D.C. three wire Transmission Net-work.**

THE METHOD

In the method described in this paper the heating effects of steady currents are considered. An electrical theorem in connection with the heating effects is used (Southwell and Black, 1938) and it can be enunciated as follows :

"In a network of conductors to which specified currents are supplied at two or more nodal points the actual distribution of currents is such that the total generation of heat less twice the energy expended in supplying the specified currents from a source at datum potential has its minimum value consistent with the satisfaction of Kirchoff's second law."

Two nodal points A and G of the network shown in Fig. 2, are considered and they are joined by a conductor of resistance R_{AG}. By Ohm's law a current of $\frac{V_A-V_G}{R_{AG}}$ will flow from A to G, (V_A and V_G are the potentials at A and G). If I_A and I_G be the currents flowing towards A and G respectively then $-I_A = I_G = g_{AG}(V_A-V_G)$, where $g_{AG} = 1/R_{AG}$. When all the conductors connected to A are considered, it can be put as,

$$\sum_A g_{AG}(V_A-V_G)+I_{AO} = 0 \qquad \ldots \quad (1)$$

I_{AO} denoting the current supplied to A from outside. Then the heat generated in AG is $g_{AG}(V_A-V_G)^2$ and the total heat generated in the network can be written as,

$$2H = \sum_m g_{AG}(V_A-V_G)^2 \qquad \ldots \quad (2)$$

$\sum_m$ meaning the summation extending to every conductor. Also the rate of expenditure of energy is measured by $I_{AO}(V_A-V_O)$, if the current I_{AO} is supplied to A from an outside source at the datum potential V_O. Then the total expenditure of energy is given by,

$$\sum_n\{I_{AO}(V_A-V_O)\} = -E \qquad \ldots \quad (3)$$

$\sum_n$ denoting the summation extending to every nodal point. Hence the equation (1) is typical of the conditions for a minimum value of the quantity,

$$Q = H+E = \tfrac{1}{2}\sum_m\{g_{AG}(V_A-V_G)^2\}+\sum_n\{I_{AO}(V_A-V_O)\} \qquad \ldots \quad (4)$$

because it is equivalent to,

$$-\frac{\partial Q}{\partial V_A} = -\frac{\partial}{\partial V_A}(H+E) = 0$$

Due to the presence of the source of E.M.F. (generators) the problem has to be modified as indicated below and then can be solved easily by relaxation method using the above theorem. It can be modified by assuming that the whole E.M.F. of each source is utilized to pass currents to earth through its own resistance and the datum distribution of known currents to enter and to leave the network at nodal points is obtained Then the effects of neutralising currents are simply to be calculated and superposed at those points.

Considering only the source of E.M.F. 'E_1' let A and F be supposed to be joined by a wire of zero resistance as shown by dotted line. Then the current passing through that source from A to F would return by that wire and hence in the datum distribution a current of $\frac{E_1}{R_{G1}}$ enters the system at A and leaves at F, R_{G1} being the internal resistance of the source (generator). Next the current distribution is to be calculated and superposed which will result when the neutralising currents $+\frac{E_1}{R_{G1}}$ and $-\frac{E_1}{R_{G1}}$ are supplied at F and A after the source of E.M.F. is removed.

Considering both the generators and all the branches of the network shown in Fig. 2, the expression for Q and the residuals can be written with the help of equation (4) as given below:

$$2Q = \frac{(V_G-V_A)^2}{R_{AG}} + \frac{(V_G-V_F)^2}{R_{FB'G}} + \frac{(V_A-V_F)^2}{R_{G1}} + 2\frac{E_1}{R_{G1}}\{V_O-V_F-(V_O-V_A)\}$$

$$+\frac{(V_G-V_F)^2}{R_{FC'G}} + \frac{(V_G-V_D)^2}{R_{DG}} + \frac{(V_F-V_D)^2}{R_{G2}} + 2\frac{E_2}{R_{G2}}\{V_O-V_D-(V_O-V_F)\} \quad \ldots \quad (5)$$

Hence,

$$\left.\begin{aligned}
-\frac{\partial Q}{\partial V_G} &= -\frac{V_G-V_A}{R_{AG}} - \frac{V_G-V_F}{R_{FB'G}} - \frac{V_G-V_F}{R_{FC'G}} - \frac{V_G-V_D}{R_{DG}} = 0 = F_G \\
-\frac{\partial Q}{\partial V_A} &= \frac{V_G-V_A}{R_{AG}} - \frac{V_A-V_F}{R_{G1}} - \frac{E_1}{R_{G1}} = -\frac{E_1}{R_{G1}} = F_A \\
-\frac{\partial Q}{\partial V_F} &= \frac{V_G-V_F}{R_{FB'G}} + \frac{V_A-V_F}{R_{G1}} + \frac{V_G-V_F}{R_{FC'G}} - \frac{V_F-V_D}{R_{G2}} = 0 = F_F \\
-\frac{\partial Q}{\partial V_D} &= \frac{V_G-V_D}{R_{DG}} + \frac{V_F-V_D}{R_{G2}} + \frac{E_2}{R_{G2}} = \frac{E_2}{R_{G2}} = F_D
\end{aligned}\right\} \quad (6)$$

If these residuals F_G, F_A, F_F and F_D obtained initially are liquidated, the potentials at the points A, G, F and D are found out and the corresponding currents can be calculated from them knowing the required resistances of different branches. In order to liquidate them the basic unit, group operation and the relaxation tables (Table I and II) are prepared (Allen, 1954; Dutta, 1966). All these procedures are elaborately shown in the following illustration.

The following example worked out by Christie (1952) using different method is taken for illustration.

In the D.C. Three Wire Transmission System shown in Fig. 1, the E.M.F. of the generators are $E_1 = E_2 = 110$ volts, the internal resistances of the generators are $R_{G1} = R_{G2} = 0.2$ ohm, the resistance of the neutral wire is $R_{FG} = 0.4$ ohm, the outer line resistances are $R_{AB} = R_{DC} = 0.1$ ohm, the load resistances are $R_{BG} = 8.0$ ohms and $R_{GC} = 10.0$ ohms. The currents flowing in the two outer and the neutral wires are to be found out.

From the supplied data, the values of the resistances of different branches of the network shown in Fig. 2, are $R_{AG} = R_{AB}+R_{BG} = 8.1$ ohms, $R_{DG} = R_{DC}+ R_{CG} = 10.1$ ohms, $R_{FC'G} = R_{FB'G} = 2\times R_{FG} = 0.8$ ohm.

With the substitution of the numerical values in the relation (5) and (6), it can be written as follows :

$$2Q = \frac{(V_G-V_A)^2}{8.1} + \frac{(V_G-V_F)^2}{0.8} + \frac{(V_A-V_F)^2}{0.2} + 2\times 550(V_A-V_F) + \frac{(V_G-V_F)^2}{.0.8}$$

$$+\frac{(V_G-V_D)^2}{10.1} + \frac{(V_F-V_D)^2}{0.2} + 2\times 550(V_F-V_D)$$

or
$$2Q = \frac{(V_G-V_A)^2}{8.1} + \frac{(V_G-V_F)^2}{0.4} + \frac{(V_A-V_F)^2}{0.2} + \frac{(V_G-V_D)^2}{10.1} + \frac{(V_F-V_D)^2}{0.2}$$
$$+2\times 550(V_A-V_F)+2\times 500(V_F-V_D) \qquad \ldots \text{(5a)}$$

$$\left.\begin{aligned}
-\frac{\partial Q}{\partial V_G} &= -\frac{V_G-V_A}{8.1} - \frac{V_G-V_F}{0.4} - \frac{V_G-V_D}{10.1} = 0 = F_G \\
-\frac{\partial Q}{\partial V_A} &= \frac{V_G-V_A}{8.1} - \frac{V_A-V_F}{0.2} - 550 = -550 = F_A \\
-\frac{\partial Q}{\partial V_F} &= \frac{V_G-V_F}{0.4} + \frac{V_A-V_F}{0.2} - \frac{V_F-V_D}{0.2} = 0 = F_F \\
-\frac{\partial Q}{\partial V_D} &= \frac{V_G-V_D}{10.1} + \frac{V_F-V_D}{0.2} + 550 = 550 = F_D
\end{aligned}\right\} \qquad \ldots \text{(6a)}$$

On liquidating the residuals the potentials at different nodal points are obtained from the relaxation table. Then the required currents can be easily calculated from those values of potentials and the resistances of the outer and neutral wires as given below :

$V_{GA} = 106.4280$ volts; where V_{GA} is the potential of G with respect to A,
$V_{DG} = 108.7914$ " ; ...... V_{DG} .... D ...... G,
$V_{FG} = 0.9432$ " ; ...... V_{FG} .... F ...... G,
$I_{GA} = 13.13$ amps; where I_{GA} is the current flowing in the wire joining G and A,
$I_{DG} = 10.78$ " ; ...... I_{DG} .... D ...... G,
$I_{FG} = 2.46$ ...... I_{FG} G,

TABLE I
Operational Table

Operation number	δV_G	δV_A	δV_F	δV_D	δF_G	δF_A	δF_F	δF_D
Unit Operation (1)	1	–	–	–	−2.7224	0.1234	2.5	0.099
(2)	–	1	–	–	0.1234	−5.1234	5.0	0
(3)	–	–	1	–	2.5	5.0	−12.5	5.0
(4)	–	–	–	1	0.099	0	5.0	−5.099
Group Operation (5) [(2)× −1+(3)× −1.0247+(4)× −1.5617]	—	−1	−1.0247	−1.5617	−2.8397	0	0	2.8397

TABLE II
Relaxation Table

	δV_G	δV_A	δV_F	δV_D	F_G	F_A	F_F	F_D
	$V_G=V_A=V_F=V_D=0$				0	−550	0	550
(*a*) [(3)×110]	—	—	110	—	275	0	−1375	1100
(*b*) [(4)×275]	—	—	—	275	302.225	0	0	−302.225
(*c*) [(5)×106.428]	—	−106.4280	−109.0568	−166.2086	0	0	0	0
	0	−106.4280	0.9432	108.7914	0	0	0	0

The values of the currents in the wires GA, DG and FG thus found by relaxation method are compared with those calculated by conventional method of network analysis and they are found to be in good agreement as shown in the (Table III) below :

TABLE III

Comparison of values

Unknown quantities	Values calculated by	
	Relaxation Method	Conventional Method
I_{GA}	13.13 amps	13.30 amps
I_{DG}	10.78 amps	10.90 amps
I_{FG}	2.46 amps	2.40 amps

DISCUSSION

This method is seen to be a convenient one for having the values of all the unknown quantities obtained simultaneously, such as the potentials at the nodal points in the problem. With the increase of the number of branches of the network in the electrical problems, the conventional methods become laborious whereas this method can yield much quicker solution preferably with little practice of relaxation technique. In this method the internal resistances of the generators or the resistances of the paths AF and FD are to be known in order that the current in the datum distribution may be calculated.

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